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An information fusion approach based on weight correction and evidence theory

resistance.

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1. Introduction

Decisions may have to be made by integrating information from multiple sources in complex environments, while these sources can be inconsistent with each other due to, e.g. incompleteness and uncertainty of information, subjectivity of each decision agent, etc. This makes it a challenging issue to reasonably fuse conflicting evidence in information fusion.

Dempster-Shafer theory [\[1,2\]](#page-8-0), as an uncertainty inference method and a general extension [\[3\]](#page-8-0) of Bayesian theory, has been widely used in data fusion $[4,5]$, debate reasoning $[6]$, pattern recognition $[7,8]$, etc. due to its capability of handling imprecise information [\[9\]](#page-8-0) and dealing with conflicts. In Dempster-Shafer theory, reasoning is performed by aggregating independent sources of evidence from different sources, but the results of evidence fusion according to the combination rules are often unsatisfactory for highly conflicting evidence sources, or even completely contradict the subjective opinions [\[10,11\].](#page-8-0) One group of scholars directly improves on the Dempster-Shafer theory's rules of combination. Yager [\[12\]](#page-8-0) assigns the conflicting part of the evidence to the recognition framework, but greatly reduces the probability value assigned to the focal element. Dubois and Prade [\[13\]](#page-8-0) assign the conflicting evidence to the concatenation of conflicting focal elements, but the method undermines the Dempster-Shafer theory combination rule of the union law. Lefevre [\[14\]](#page-8-0) put forward a general framework for conflict redistribution after integrating multiple combination rules. Mihai Cristian Florea [\[15\]](#page-8-0) and Yee leung [\[16\]](#page-8-0) use the analysis and combination rules to distribute some of the conflicts to all groups in a certain proportion. Gao [\[17\]](#page-8-0) and Deng [\[18\]](#page-8-0) use quantum theory to simulate the uncertainty of mass function. Another group directly modifies the sources of evidence. Murphy [\[19\]](#page-8-0) performs weighted averaging of sources of evidence before combining them. Deng [\[20\]](#page-8-0) gives a method for calculating the weights of sources of evidence by means of the evidential distances proposed by Jousselme [\[21\].](#page-8-0) Jiang [\[22\]](#page-9-0) fuses the conflict k and the Jousselme distance to jointly represent the evidential conflicts, but does not give the weight calculation of the fused two. Fei [\[23\]](#page-9-0) adopted the evidence best-worst method and combined it with DST to make up for the limitation of the traditional weight calculation method in expressing uncertainty. Liu and Xiang [\[24\]](#page-9-0) form a composite discount factor to correct the body of evidence based on improved Shafer conflict metric formula, Jousselme distance, trust entropy and

show that the proposed rules can manage conflicting evidence with better convergence and interference

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Received 30 October 2023; Received in revised form 22 June 2024; Accepted 11 October 2024 Available online 28 October 2024 1877-7503/© 2024 Elsevier B.V. All rights are reserved, including those for text and data mining, AI training, and similar technologies. game theory. Huang [\[25\]](#page-9-0) proposes an enhanced belief log-log similarity measure based on DST that better takes into account the internal differences of subsets. Hamda [\[26\]](#page-9-0) utilized Hellinger's distance and Deng's entropy, a measure of conflict and uncertainty in evidence, to assess the reliability of evidence and its weights, reducing the impact of unreliable evidence. Although several methods and alternatives have been proposed in the literature, the first category of methods would destroy the exchange and combination properties of the classical Dempster combination rule, and how to effectively resolve conflicts between evidence is still an open question for the second category of methods that needs further improvement.

In this paper, an improved evidence combination rule based on adaptive weight correction is proposed to improve the ability of Dempster-Shafer theory to manage high conflict evidence. The method defines two new metrics: (i) the degree of focus dispersion and (ii) preference consistency. The method aims to assess the importance of each source of evidence in preprocessing by two metrics. The main idea is to assign a high weight to sources with more focused information and higher degree of consistency and a low weight to sources with more dispersed information and lower degree of consistency. This strategy aims to reduce the conflicting influence of sources with more dispersed information or higher degree of consistency angle on the final combination results, thus improving the system's ability to manage conflicting evidence.

The main contributions of this study are summarized as follows:

- − We first propose a measure of evidence focal element dispersion, which can effectively quantify the degree of concentration of the information distribution of the evidence sources to improve the combining strategy of conflict coefficient k and Jousselme distance in weight correction.
- − Defined a metric of evidence preference consistency, which can effectively quantify the degree of tendency consistency of each piece of evidence, when the degree of consistency is higher we will increase the credibility of the evidence to improve its weight.
- − Finally, we apply the proposed method to some classical examples as well as Monte Carlo simulations to demonstrate the good performance compared to other typical combination rules. The results show the superiority of the rules proposed in this paper in terms of conflict evidence management, convergence speed, anti-interference ability and accuracy of decision making.

The rest of the paper is composed as follows: Section 2 introduces some basic theories, including Dempster-Shafer theory, a weight correction rule proposed by Jiang [\[22\]](#page-9-0). [Section 3](#page-2-0) introduces the concepts of Lai Jiao Yuan discretization and preference consistency and their metric formulas as a way to make improvements to the weight correction rule. [Section 4](#page-4-0) presents and discusses specific examples comparing the improved weight correction rule with some classical rules. Finally, we summarize the whole paper in [Section 5](#page-6-0).

2. Dempster-Shafer theory of evidence

Evidence theory is an uncertainty-based reasoning method that demonstrates the degree of uncertainty of information and fuses multiple sources of information for the purpose of reasoning and decision making. To facilitate the discussion, this section first briefly introduces the basic concepts of the theory and the problems of the combination rules, and then proposes a new rule of evidence fusion by modifying the evidence sources.

2.1. Basic concepts

In evidence theory, the uncertainty of information is determined by the identification framework and the probability distribution function. Assuming that $\theta = {\theta_1, \theta_2, ..., \theta_n}$ is a finite nonempty set containing all single hypotheses of the target problem and that the hypotheses are independent of each other. *Θ* is called the identification framework and 2*^Θ*is the power set of *Θ*, denoting the set consisting of all subsets of *Θ*. In the specific problem, the plausibility of any proposition can be represented by a subset of the identification framework.

The Basic Probability Assignment function (BPA) m is represented as a mapping of the power set 2*^Θ*to [0,1], which assigns probability values to all subsets of the recognition framework, while the function m satisfies the following two conditions:

$$
m(\emptyset) = 0 \tag{1}
$$

$$
\sum_{A \subseteq \Theta} m(A) = 1 \tag{2}
$$

Let *A* be an element of the power set. The basic probability assignment of the set *A* can be expressed as *m*(*A*), which can be interpreted as the probability of supporting a true subset of *Θ* that belongs to the set *A* but not to a true subset of the set *A*, where the probability of the empty set is 0 and the sum of the probabilities of all subsets in *Θ* is 1. The value of $m(A)$ is expressed as the plausibility of the proposition *A*. If $m(A) = 0$ or $m(A) = 1$ mean that the proposition *A* is completely implausible or completely plausible, respectively, and if $m(A) > 0$, the proposition *A* is said to be the focal element (focal element).

For an identification framework *Θ*, two other important metric functions based on the basic probabilistic distribution function m are the confidence function *Bel*(*A*) and the likelihood function *Pl*(*A*):

$$
Bel(A) = \sum_{B \subseteq A} m(B) \tag{3}
$$

$$
Pl(A) = \sum_{A \cap B \neq \emptyset} m(B) \tag{4}
$$

 $Bel(A)$ and $Pl(A)$ are the minimum uncertainty and maximum uncertainty of the set *A*, respectively. [*Bel*(*A*)*, Pl*(*A*)] can be used to denote the uncertainty of the set *A* degree of confidence interval, and the relationship between them is as follows:

$$
Pl(A) = 1 - Bel(A) \tag{5}
$$

$$
Pl(A) \geq Bel(A) \tag{6}
$$

where \overline{A} is the complement of *A*. If the difference between *Bel*(*A*) and *Pl*(*A*) is too large, the reliability of this information source is low.

2.2. Combination rules

When there are multiple independent sources of evidence, the Dempster-Shafer theory's evidence combination rule is:

$$
m(A) = \begin{cases} \sum_{\substack{\cap A_{j} = A \\ 0 & A = \emptyset}} \prod_{i=1}^{n} m_{i}(A_{i}) \\ 1 - k & A \neq \emptyset \end{cases}
$$
(7)

where n denotes the number of independent sources of evidence, and $m_i(A_i)$ denotes the underlying probability assignment value of A_i under the ith source of evidence, and *k* denotes the degree of conflict among the n independent sources of evidence:

$$
k=\sum_{\substack{\cap A_j=0}}\prod_{i=1}^n m_i(A_j) \hspace{1cm} (8)
$$

k is the probability of the empty set before normalization when evidence is fused, called the conflict coefficient, which represents the degree of conflicting information among evidence sources. *k* is between the interval [0*,*1], and a larger value of *k* indicates a higher degree of conflict among evidence sources.

There is an unavoidable problem with Dempster'rule. When the

evidence is completely conflicting, i.e. when $k = 1$, the value of the denominator 1 − *k* used for normalization in the combination rule is 0, and the formula cannot be used for combination at that time. In addition, when the evidence is highly conflicting, i.e. when $k \rightarrow 1$, the results of using this combination rule are often unrealistic and counterintuitive.

Example 1. Consider the case of highly conflicting evidence, which we illustrate using a classic counterexample proposed by Zadeh [\[10\]](#page-8-0). Suppose there exist two independent sources of evidence m_1 and m_2 under the same identification framework $\theta = {\theta_1, \theta_2, \theta_3}$ such that the structure of the two sources is as follows:

 ${m_1 : m_1(\theta_1) = 0.9, m_1(\theta_2) = 0.1}$

 $m_2 : m_2(\theta_2) = 0.1, m_2(\theta_3) = 0.9$

The results of evidence fusion using the D-S evidence combination rule are: $m(\theta_1) = 0, m(\theta_2) = 1, m(\theta_3) = 0$. It is easy to see that both m_1 and m_2 provide low support for hypothesis θ_2 , but the fusion result is fully supportive of hypothesis θ_2 . In addition, m_1 and m_2 provide high support for hypotheses θ_1 and θ_3 . However, the fusion result is not supportive of these two hypotheses. This fusion result is seriously counterintuitive.

3. Adaptive weighting correction rule

3.1. The weighting correction rule

In order to avoid the problem that the Dempster'rule cannot handle highly conflicting evidence, a class of weighting correction rule has been proposed [\[22\]](#page-9-0) to use the conflict coefficient and Jousselme distance as indicators of the degree of conflict between evidence sources, and by the degree of conflict with other evidence, the weights of each evidence source can be determined for weighted averaging to modify the evidence sources. When both indicators are larger, the evidence is more conflicting with other evidence, less similar and less credible, and therefore is assigned a smaller weight, and vice versa. We call this rule the WCR, where the weights among the n evidence sources are calculated as follows.

Let m_1 and m_2 be two BPAs under the identification frame Θ . Then the Jousselme distance (also called evidence distance) between m_1 and $m₂$ is:

$$
d_{12} = \sqrt{\frac{1}{2} (||\overrightarrow{m_1}||^2 + ||\overrightarrow{m_2}||^2 - 2\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle)}
$$
(9)

where $||\vec{m}||^2 = \langle \vec{m}, \vec{m} \rangle$, $\langle \vec{m_1}, \vec{m_2} \rangle$ denotes the inner product of two vectors.

$$
\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle = \sum_{i=1}^m \sum_{j=1}^m m_1(A_i) m_2(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|}
$$
(10)

where m denotes the power set 2*^Θ*of elements, *A* denotes an element of 2*^Θ*, and |*A*| is called the potential of *A*, i.e. the number of elements contained in the set *A*, *m* denotes the number of elements of the power set 2*^Θ*.

Let there be a total of n evidence sources, the two-two evidence distance d and the conflict coefficient k of all evidence sources can be calculated by Eqs. (8) and (9) , and the conflict degree c is obtained by averaging the two indicators and expressed as the conflict degree matrix *CM*:

$$
CM = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{nn} \end{bmatrix}
$$
 (11)

$$
c_{ij} = \frac{d_{ij} + k_{ij}}{2} \tag{12}
$$

where *CM* is the symmetric matrix and *cij* denotes the conflict degree between m_i and m_j . When the conflict degree is smaller, the similarity

between evidence is larger, so define the similarity between m_1 and m_2 as *Sim*12:

$$
Sim_{12} = 1 - c_{12} \tag{13}
$$

Based on the distance matrix *CM*, the similarity matrix *SM*can be obtained as follows:

$$
SM = \begin{bmatrix} sim_{11} & sim_{12} & \cdots & sim_{1n} \\ sim_{21} & sim_{22} & \cdots & sim_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ sim_{n1} & sim_{n2} & \cdots & sim_{nn} \end{bmatrix}
$$
 (14)

When the similarity between two sources of evidence is greater, it means that the credibility of these two sources of evidence is higher, so the similarity between m_i and other evidence can be used to express the support of other evidence for *mi*, denoted as *Supi*:

$$
Sup_i = \sum_{j=1, j \neq i}^{n} Sim_{ij}
$$
\n(15)

The credibility of m_i can be expressed quantitatively by normalizing the support of *mi* by other evidence, denoted as *Crdi*:

$$
Crd_i = \frac{Sup_i}{\sum_{i=1}^{n} Sup_i}
$$
 (16)

It is not difficult to find that when the conflict coefficient k and the evidence distance d between *mi* and other evidence are smaller, the higher the credibility and the greater the weight. The weighted average of the evidence sources by Murphy's method [\[19\]](#page-8-0) yields the weight-corrected evidence sources $m_{WAF}(A)$:

$$
m_{\text{WAE}}(A) = \sum_{i=1}^{n} \text{Crd}_{i} \cdot m_{i}(A) \qquad (17)
$$

where *A* is an element of the power set 2*^Θ*. The corrected evidence $m_{WAE}(A)$ is combined n-1 times with Dempster' rule to be the final fused evidence. The weight calculation process of WCR is shown in [Fig. 1.](#page-3-0)

When the source reliability of the evidence is unknown, the above rule can be used to correct the evidence by assigning weights to the evidence in terms of interrelationships between the evidence. However, in the process of calculating the degree of conflict, the rule always uses the same proportion to fuse the two indicators of conflict coefficient and evidence distance. When there is consistency between these two indicators, the degree of conflict between evidence can be calculated effectively. When dealing with some special evidence sources, the two will be inconsistent, and at this time, if the two indicators are fused with the same proportion, the obtained conflict degree does not give a reasonable explanation.

Example 2. Considering the case of inconsistent conflict coefficients and evidence distance indicators, assume the existence of two sets of evidence, each containing two independent sources of evidence under the same identification framework $\theta = {\theta_1, \theta_2, \theta_3}$ such that the struc-

ture of the two sets of evidence sources is as follows.
\n
$$
\begin{cases}\nm_1: m_1(\theta_1) = 0.4, m_1(\theta_2) = 0.3, m_1(\theta_3) = 0.3 \\
m_2: m_2(\theta_1) = 0.4, m_2(\theta_2) = 0.3, m_2(\theta_3) = 0.3 \\
m_3: m_3(\theta_1) = 0.8, m_3(\theta_2) = 0.1, m_3(\theta_3) = 0.1 \\
m_4: m_4(\theta_1) = 0.8, m_4(\theta_2) = 0.1, m_4(\theta_3) = 0.1\n\end{cases}
$$

Calculated from [Eqs. 8 and 9](#page-1-0) we get $k_{12} = 0.66$, $d_{12} = 0$, $k_{34} = 0.34$ and $d_{34} = 0$. It can be found that the two metrics do not vary consistently under different groups, and the credibility of the results cannot be explained if both metrics are given a weight of 0.5 to find the degree of evidence conflict under these two different groups of evidence. The reason for the inconsistency of the metrics is the inconsistency of the discrete characteristics of the focal elements of the evidence itself. Although the basic probability assignments of the two evidence sources in each group are exactly the same, because the probability assignments of the second group are more concentrated compared with the first

Fig. 1. WCR's weight calculation process.

group (both assign a larger mass to θ_1), the degree of focal element dispersion is smaller, resulting in a smaller conflict coefficient in the second group. Therefore, when the two metrics are fused, when the degree of focal element dispersion of the evidence is smaller, the conflict coefficient indicator *k* should theoretically be assigned a larger weight and the evidence distance metric *d* should be assigned to a smaller weight to balance the influence of the two metrics in the matrix from the dynamics.

3.2. Concept of adaptive weighting correction rule

Regarding the weighting correction, we have already pointed out in Example 2 that the traditional weighting correction does not take into account the effect of the degree of dispersion of focal elements. From an informational perspective, when a source is more focused in the assignment of evidence focal elements, the more valid the information contained in the source is, and the less discrete the focal elements within the evidence are at that time. Conversely, when the assignment of focal elements in an evidence source is relatively balanced, the degree of uncertainty of that evidence is higher. In the following, we introduce information entropy to measure the dispersion of focal elements within the evidence, and automatically balance the weights of conflict coefficient *k* and evidence distance *d* according to the dispersion of focal elements of the evidence to be fused. We call this rule the adaptive weight correction rule, referred to as AWCR.

Definition 1. Let *mi* be a BPA under the recognition frame *Θ*. The power set 2*^Θ*of this recognition frame has n elements, then the degree of focal element dispersion can be defined as:

$$
- \sum_{j=1}^{n} m_i(\theta_j) \ln(m_i(\theta_j))
$$

$$
Df e_i = \frac{m_i(\theta_j) \neq 0}{\ln(n)}
$$
 (18)

The information entropy increases as the uncertainty of the evidence increases, and obviously *Dfe_i* is an increasing function of the uncertainty of *mi*. The information entropy is minimized to − *ln*(1) = 0 when a focal element in the evidence is assigned the full mass, and maximized to 1 when the mass of each focal element of the evidence is 1/n. Thus by normalization of the maximum information entropy, the $Dfe_i \in [0,1]$.

Definition 2. Let m_i and m_j be two BPAs under an identification framework *Θ*, the degree of conflict between two sources of evidence can be defined by making an improvement to Eq. (12) as:

$$
K_{ij} = w_{ij}^k k_{ij} + w_{ij}^d d_{ij}
$$
 (19)

where w_{ij}^k and w_{ij}^d denote the conflict degree weighting factor and evidence distance weighting factor between *mi* and *mj* based on the degree of dispersion of focal elements:

$$
w_{ij}^k = 1 - \frac{Dfe_i + Dfe_j}{2} \tag{20}
$$

$$
w_{ij}^d = \frac{Dfe_i + Dfe_j}{2} \tag{21}
$$

where w_{ij}^k denotes the total focal element dispersion of m_i and m_j taking values in the range [0,1], it is obvious that $w_{ij}^k + w_{ij}^d = 1$. And it is consistent with the ideal behavior of adaptive weight correction: as the focal element dispersion of the two evidence sources to be fused increases, the influence of the conflict coefficient k on the degree of conflict decreases accordingly, and the influence of the evidence distance d increases accordingly, so that w_{ij}^k is a decreasing function of the total focal element dispersion and w_{ij}^d is an increasing function. The weight calculation process of AWCR is shown in Fig. 2.

1)When
$$
Dfe_i + Dfe_j \rightarrow 0
$$
,
\n $w_{ij}^k \rightarrow 1$, $w_{ij}^d \rightarrow 0$, $K_{ij} \rightarrow k_{ij}$
\n2) When $Dfe_i + Dfe_j \rightarrow 2$,
\n $w_{ij}^k \rightarrow 0$, $w_{ij}^d \rightarrow 1$, $K_{ij} \rightarrow d_{ij}$

Fig. 2. AWCR's weight calculation process.

3) When
$$
Dfe_i + Dfe_j \rightarrow 1
$$
,
\n $w_{ij}^k \rightarrow 0.5$, $w_{ij}^d \rightarrow 0.5$, $K_{ij} \rightarrow \frac{k_{ij} + d_{ij}}{2}$

It can be seen that the conflict coefficient *k* and the evidence distance *d* and the averaging treatment correspond to three special cases in this rule where the total focal element dispersion is too small, too large and appropriate.

3.3. AWCR with preference consistency

ARCR only considers the degree of focal element dispersion of the evidence to be fused. However, in a piece of evidence, its largest focal element is the most critical focal element of this evidence, which contains the largest amount of information about this evidence and represents the overall preference of the evidence. Therefore, we add another measure of evidence bias, preference consistency, to redistribute the evidence weights of AWCR, and we call this rule AWCR1(AWCR with preference consistency).

Definition 3. With n sources of evidence m_1, m_2, \ldots, m_n from the same identification frame *Θ* under which the power set 2*^Θ*has a total of m elements, there exists a maximum focal element location matrix defined as *P*:

$$
P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1m} \\ p_{21} & p_{22} & \cdots & p_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nm} \end{bmatrix}
$$
 (22)

$$
p_{ij} = \begin{cases} 1ifj \in pos(max(m_i)) \\ 0else \end{cases}
$$
 (23)

Where $pos(max(m_i))$ denotes the position of $max(m_i)$, i.e. the set of positions consisting of the element indices of all maximal focal elements of *mi* in the power set 2*^Θ*. According to the matrix *P*, the preference consistency of each evidence source can be measured as follows.

1) When m_i has only one maximum focal element.

$$
Pre_i = \sum_{x=1}^n p_{xy} \tag{24}
$$

Where $y = pos(max(m_i))$, the set $pos(max(m_i))$ contains only one element since there is only one maximal focal element in m_i at this point. 2) When m_i has more than one maximum focal element.

$$
Pre_i = \frac{1}{|max(m_i)|} \sum_{y \in pos(max(m_i))} \sum_{x=1}^{n} p_{xy}
$$
 (25)

where $|\max(m_i)|$ denotes the potential of $\max(m_i)$, i.e. the maximum number of focal elements of m_i . *Pre_i* is the preference consistency of m_i . The position of the largest focal element in the evidence represents the preference of that evidence. When there are more evidence with the same preference, their preference consistency is higher, and their credibility is higher compared with other evidence, and they should be assigned higher weights. When *mi* has only one maximal focal element, its preference consistency is expressed as the sum of the values of the columns where this focal element is located in the matrix *P*. When *mi* has more than one maximal focal element, its simultaneous existence of multiple preferences, the consistency of its individual preferences is expressed as the sum of the values of the columns in which each maximal focal element is located in the matrix *P*, and its preference consistency is expressed as the average of the consistency of each preference. It is easy to see that Eq. (24) is a special case of Eq. (25) , so the preference consistency of all evidence can be represented by Eq. (25) .

Definition 4. Let *mi* be a BPA under the identification framework *Θ*, which has a power set 2^{*Θ*} with n elements. Making improvements to Eq. [\(16\),](#page-2-0) the confidence level of m_i can be defined as:

$$
Crd_i = \frac{Sup_i \cdot Pre_i}{\sum_{i=1}^{n} Sup_i \cdot Pre_i}
$$
 (26)

Obviously, *Crdi* is an increasing function of Pr*ei*. As the evidence preference consistency increases, its credibility increases accordingly, which is consistent with the ideal behavior of preference consistency. the weight calculation process of AWCR1 is shown in [Fig. 3](#page-5-0). In particular, the computation of the conflict coefficient *k* and evidence distance *d* requires the computation of n evidence sources two by two with a time complexity of $O(n^2)$, and the computation of the degree of focal element discretization involves the analysis of the BPA of each evidence source with a time complexity of $O(n)$. The calculation of preference consistency involves identifying and comparing the maximum focal elements of each evidence source with a time complexity of $O(n)$. Weight calculation adjusts the weight of each evidence source based on the conflict degree, focus element dispersion and preference consistency, the time complexity of this process is proportional to the number of evidence sources and can be considered as $O(n)$. Finally, according to the adjusted weights, the evidence is fused using Dempster's combination rule, although the Dempster-Shafer theory has a potential exponential explosion problem, this method mainly preprocesses the evidence sources before fusion, so the time complexity of the two is independent of each other, and does not suffer from the Dempster-Shafer theory in the preprocessing process exponential explosion problem. Therefore, the total time complexity of this method is $O(n^2)$.

3.4. Summary of this chapter

In this chapter, we propose two metrics, focal element dispersion degree and preference consistency, to improve the WCR. In the next section, we compare the fusion results of AWCR and other existing rules under the evidence sources described in the counterexample proposed by Zadeh (i.e. Example 1) and verify the advantages of our improved rule using several examples.

4. Numerical examples

4.1. Combination of two pieces of evidence

Example 3. Based on the example in [\[10\]](#page-8-0), assume the existence of two independent pieces of evidence m_1 and m_2 under the same identification framework $\theta = {\theta_1, \theta_2, \theta_3}$ such that the two sources of evidence are structured as follows.

$$
\begin{cases}\nm_1 : m_1(\theta_1) = \beta, m_1(\theta_2) = \alpha, m_1(\theta_3) = 1 - \alpha - \beta \\
m_2 : m_2(\theta_1) = 1 - \alpha - \beta, m_2(\theta_2) = \alpha, m_2(\theta_3) = \beta\n\end{cases}
$$

Where $0 \le \alpha \le 1$, $0 \le \beta \le 1$ and $0 \le \alpha + \beta \le 1$, when $\alpha = 0.1$ and $\beta = 0$, Example 3 becomes Example 1. [Table 1](#page-5-0) summarizes the results obtained by applying the improved rules of this paper and some classical rules to Example 1. Since there are only two evidences for fusion, both evidence weights are 0.5 in both WCR and AWCR, and since the preference consistency of both evidence sources are equal to 1, AWCR1 does not make any modification to the weights in the pair reassignment process and obtains the same results as WCR and AWCR. All the above three rules solve the 1 trust paradox problem in Dempster'rule, and the final fusion results all assign a smaller mass to the focal element θ_2 , which is consistent with the intuitive human judgment.

To assess the resistance of various evidence fusion rules to interference at different levels of conflict, let $\alpha = 0.1$, the change in β is used to modulate the degree of conflict between m_1 and m_2 . Since the preference consistency of the two evidence sources is always equal as *β* varies and CM is a symmetric matrix, Thus WCR, AWCR and AWCR1 always have the same results in two-evidence fusion, only the fusion results for Dempster's rule and AWCR are shown in [Fig. 4.](#page-5-0)

[Fig. 4](#page-5-0) presents the sensitivity of the two rules under different

Fig. 3. AWCR1's weight calculation process.

Table 1 Fusion of two evidence m_1 and m_2 in Example 1.

Focal element	Dempster	Yager ___	WCR ____	AWCR	AWCR1
θ_1			0.488	0.488	0.488
θ_2		0.010	0.024	0.024	0.024
θ_3			0.488	0.488	0.488
Θ		0.990	0		0

conflicts. It is not difficult to find that the fusion results of AWCR1 are always more reasonable, assigning larger masses to focal elements θ_1 and θ_3 in Fig. 4a. In Fig. 4b, The mass of focal element θ_2 is almost zero and does not change significantly with the conflict. However, in the Dempster'rule, as the degree of conflict increases the mass of each focal element changes more and more significantly, focal elements θ_1 and θ_3 gradually decrease and eventually converge to 0 in Fig. 4a, while focal element θ_2 gradually increases and eventually converges to 1 in Fig. 4b, giving rise to the 1 trust paradox problem.

4.2. Combination of evidence from multiple sources

In the combination of the two evidence, the weight of the two evi-

dence sources is always 0.5 due to the symmetry of the conflict degree matrix *CM*, which cannot change the weight correction value even considering the focal element dispersion degree metric, and the preference consistency metric of the two evidence sources also always remains the same and cannot have an impact on the weight correction. To further demonstrate the positive impact of these two metrics in the weight correction process, in the follow-up we use a multiple evidence fusion example to compare the results of each rule.

Example 4. We analyze the role of two metrics, focal element dispersion and preference consistency, using a comprehensive numerical example of an automatic multi-sensor-based target recognition system proposed by [\[27\].](#page-9-0) Suppose there exist 7 independent evidence under the same recognition framework $\theta = {\theta_1, \theta_2, \theta_3}$. The final target of the system is θ_1 , where m_2 is unreliable evidence, such that the structure of the 7 evidence sources is as follows.

$$
\left\{\n\begin{array}{l} m_1:m_1(\theta_1)=0.53, m_1(\theta_2)=0.15, m_1(\theta_3)=0.32\\ m_2:m_2(\theta_1)=0, m_2(\theta_2)=0.9, m_2(\theta_3)=0.1\\ m_3:m_3(\theta_1)=0.54, m_3(\theta_2)=0.06, m_3(\theta_3)=0.4\\ m_4:m_4(\theta_1)=0.57, m_4(\theta_2)=0.3m_4(\theta_3)=0.13\\ m_5:m_5(\theta_1)=0.55, m_5(\theta_2)=0.2, m_5(\theta_3)=0.25\\ m_6:m_6(\theta_1)=0.55, m_6(\theta_2)=0.1, m_6(\theta_3)=0.35\\ m_7:m_7(\theta_1)=0.56, m_7(\theta_2)=0.15, m_7(\theta_3)=0.29\n\end{array}\n\right\}
$$

According to the two metrics of conflict coefficient *k* and evidence

Fig. 4. Comparison of mass between focal element and conflict mass: (a) mass of θ_1 or θ_3 vs. conflict mass, (b) mass of θ_2 vs. conflict mass.

distance *d*, the conflict degree matrix of 7 evidence can be obtained from [Eqs. 9](#page-2-0)–12, which is the conflict degree matrix *CM* in WCR:

 CMB

In AWCR, a metric of focal element dispersion degree is introduced for the adaptive assignment of the ratio of *k* and *d*. Table 2 shows the degree of focal element dispersion for each evidence that can be calculated from [Eq. 18](#page-3-0).

According to Table 2, the conflicting degrees of evidence are redistributed from [Eqs. 20 and 21](#page-3-0) to obtain the matrix of conflicting degrees used in the AWCR for the seven pieces of evidence *CM*ʹ.

In AWCR1, a preference consistency metric was introduced to redistribute the final evidence weights, and Table 3 shows the preference consistency for each evidence source obtained from [Eqs. 22](#page-4-0)–25.

According to the matrix *CM* and *CM*ʹ , the individual evidence weight correction values of WCR and AWCR can be obtained from [Eqs. 13](#page-2-0)–16, and then the individual evidence weight correction values of AWCR1 can be obtained according to [Eq. 26](#page-4-0) and Table 3.

Fig. 5 presents the weight revision values of the seven evidences obtained under the three rules of WCR, AWCR, and AWCR1, i.e. the level of confidence in the seven evidences. Among the 7 evidences assumed in Example 4, m_2 is unreliable evidence and should theoretically be assigned a lower weight value. In Table 2 due to the smaller dispersion of the focal element of m_2 , a larger proportion is assigned to k in the adaptive assignment of the two metrics *k* and *d*, resulting in a significant increase in the second row of matrix *CM*ʹ compared to matrix *CM*, reducing the degree of trust in *m*2 and ultimately reducing the weight of m_2 . In Table 3 since m_2 is more biased towards θ_2 and the other 6 evidences are all biased towards θ_1 , the preference consistency of θ_2 is 1, which is significantly smaller than the other evidences, and after the reallocation of [Eq. 26,](#page-4-0) the weight of AWCR1 is significantly reduced. From the figure, it can be found that the weight of m_2 is ranked in the order of WCR*>*AWCR*>*AWCR1 in the three rules, i.e. the result of AWCR1 is most consistent with the theoretical behavior. [Table 4](#page-7-0)

The variation of the mass of each focal element with the number of evidence fusion under different rules is shown in [Fig. 6](#page-7-0). Dempster's rule leads to a constant mass of θ_1 in [Fig. 6](#page-7-0)a because the mass assigned to θ_1 by *m*2 is 0. Yager's rule, although it can get the correct conclusion, leads to less and less information of the fused evidence because the rule assigns the conflicting part to the identification frame. WCR, AWCR and AWCR1 all lead to the correct decision, but in [Fig. 6](#page-7-0)a and [Fig. 6](#page-7-0)b it can be found that AWCR1 has better convergence, starting from the fusion of the third evidence, and the convergence performance is significantly better than the other rules.

The final values of the seven evidences fused under different rules are shown in [Fig. 7](#page-7-0). Among them, the fusion result of Dempster's rule is

 m_1 *m*₂ *m*₃ *m*₄ *m*₅ *m*₅ *m*₆ *m*₇ w 0.5065 0.1671 0.4461 0.4866 0.5125 0.4761 0.4976

Table 2 The degree of focal element dispersion for each evidence.

Consistency of preferences across evidence.

Table 3

Fig. 5. Weight correction value distribution of each evidence under the three rules.

contrary to intuition, the fusion result of Yager's rule is not significant in terms of decision making, and the latter three rules can obtain better results, and the fusion effect is further optimized by the degree of focal element dispersion and the preference consistency metric. The accuracy of recognizing the correct target under AWCR and AWCR1 reached 98.56 % and 98.81 %, surpassing all other methods.

4.3. Monte Carlo simulation

To further demonstrate the effectiveness of evidence fusion between AWCR and AWCR1 in general, this section uses the Monte Carlo simulation proposed by [\[16\]](#page-8-0) to generate random sources of evidence for fusion. We consider the fusion of 10 independent pieces of evidence under the same identification framework $Θ = \{θ_1, θ_2, θ_3\}$. Assuming that the target of this fusion is *θ*2, we divide the evidence into two categories, one for normal evidence with preference θ_2 and the other for abnormal evidence whose preference is not θ_2 . 100 independent samples of each of these two types of evidence are randomly generated by two algorithms in [Table 5](#page-8-0) and [Table 6.](#page-8-0)

We randomly selected a certain number of two types of evidence totaling 10 from the sample for fusion. In each fusion, we randomly set the proportion of the two types of evidence among the 10 pieces of evidence, where the proportion of abnormal evidence is ≤ 50% considering that the target of this fusion is θ_2 . Thus for each experiment, the number of normal evidence *>* 5 and the number of abnormal evidence \leq 5. The resistance to interference was assessed by the mass assigned to θ_2 by the various rules. In [Fig. 8](#page-8-0), we can see that as the proportion of abnormal evidence increases, the mass of θ_2 decreases in the order of WCR *>* AWCR *>* AWCR1 *>* DS *>* Yager for the five rules. the mass of θ_2 is slightly less affected in AWCR than in WCR. compared to the other rules, AWCR1 has the strongest resistance to interference and when the proportion of interference evidence reaches 50 %, the mass of θ_2 is 0.7738, which is well above the mass mean by 1/3, with significant decision making ability.

5. Conclusion

In this paper, we demonstrate the illogical results of Dempster's rule in dealing with the high conflict problem and the information loss

Table 4 Combination results of different rules.

Fig. 6. Mass allocation of focal elements for various evidence fusions under different rules: (a) Mass allocation of θ_1 , (b) Mass allocation of θ_2 , (c) Mass allocation of *θ*3.

Number of converging evidence

problem of Yager's combination rule. For evidence conflicts, we propose to modify the weight correction rule for evidence sources by using the degree of conflict among the evidence to explain the relative reliability of the sources. If the sum of conflicts with all other evidence is low, the source is reliable, and vice versa.

For evidence conflict, a general formula for the degree of conflict is proposed based on the weighted sum of conflict coefficient k and jousselme distance. And the metric of focal element dispersion degree is defined to adjust its weight adaptively. If the total focal element dispersion degree of two evidences is higher, the influence of the conflict coefficient *k* is appropriately increased, and vice versa. In addition, the metric of preference consistency is defined to recalibrate the weights of each evidence. If the evidence shows the same preference as most of the other evidence, the reliability of the source is increased, and vice versa.

With some classical examples we compare the proposed rule in this paper with the classical rule. In the combined example of two pieces of evidence it is shown that the focal element dispersion and preference consistency metrics do not affect the weight correction results, but still Generate a normal sample of evidence in favor of θ_2 .

Table 6

Generate a sample of anomalous evidence against θ₂.

-
- 1. Set *θ*₁, *θ₂*, *θ*₃ as focal element.
2. generate a random number *x* uniformly distributed in the interval $\left|0, \frac{1}{2}\right\rangle$,and
- 3 generate a random number *y* uniformly distributed in the interval [0*,*1 − *x*].

3. Associate mass *x* with focal element θ_2 , mass *y* with focal element θ_1 or θ_3 randomly, and mass $1 - x - y$ with the remaining focal elements.

]

Fig. 8. The mass assigned to θ_2 using various rules.

give results that are more consistent with intuitive judgments than the Dempster's rule. In the combination of multiple evidence examples, it is shown that with the introduction of focal element dispersion and preference consistency metrics, especially preference consistency, the rule is more sensitive to identify anomalous evidence and assign lower weights to it, which significantly improves the convergence. Finally, by adjusting the proportion of anomalous evidence in Monte Carlo simulations, we emphasize the anti-interference ability of AWCR and AWCR1. The experimental results show that AWCR1 exhibits the resistance to interference and decision making ability due to all other rules.

CRediT authorship contribution statement

Xugang Xi: Conceptualization, Writing – review & editing. **Jian Yang:** Project administration. **Ting Wang:** Conceptualization. **Yu Zhou:** Methodology, Visualization. **Yaqing Nie:** Methodology, Visualization, Writing – original draft, Writing – review & editing. **Yun-Bo Zhao:** Writing – review & editing. **Yahong Chen:** Writing – review & editing. **Lihua Li:** Resources.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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